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A phase-type model of cohabiting union duration

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Abstract We propose a phase-type model to analyze the duration of non-conjugal cohabiting unions. This model is a compartment model with two competing events: the marriage with the partner and the separation of the couple. We suppose that a non-marital union can be characterized by two hidden phases. The first begins at the start of the union and at each moment, people have the possibility to move from this first phase to the second phase during which hazard rates of marriage and of separation differ from the first phase. Investigations on data from the British 1958 National Child Study and the 1970 British National Study show that the proposed model fits well with data about the first cohabiting union of interviewed people. Results show that processes of marriage and separation differed between the two cohorts.

1 Introduction

Our aim in this paper is to investigate cohabiting union duration with a phase-type model (Aalen, 1995; Aalen & Gjessing, 2001; Lindqvist, 2013; Lindqvist & Amundrustad, 1998). Phase-type models are duration models with two peculiarities: first, they are compartment models in which one or several states of a studied system are hidden, i.e. non-observed; second, transition hazard rates from one state to another, hidden or not, are considered to be constant (Cox, 1962; Aalen, 1995). Phase-type models are much developed in reliability theory as well as in health sciences, for example to analyze the process of degradation of a machine or

the progression of an illness (Aalen et al, 2008; Lindqvist & Amundrustad, 1998). They are potentially interesting in life course research in which notions of phases and transitions between phases are important (Levy & the Pavie team, 2005).

The model we propose to analyze cohabiting union duration is a competing risk model with two kinds of cohabitation termination: the marriage of the couple or its separation. In the next section (section 2), we sketch the interest to develop an approach of cohabiting unions with a phase-type model. In the following section (section 3), the proposed model is presented. This section is followed by a brief presentation of data we used in this paper; first, the 1958 National Child Study, and second, the 1970 British National Study (section 4). In section 5, we present and discuss our results and finally we offer a conclusion.

2 Marriage and cohabiting union formation as composed of different hidden phases

Compartment models with hidden phases are scarce in the literature of family demography. One exception is the model of first marriage that was proposed by Coale and McNeil in the early 1970s (Coale & McNeil, 1972). In this model, it is considered that people progress in a succession of different social states before the marriage: access to the marriage market at the end of adolescence, period of partner search, dating, and engagement (figure 1). All these intermediary states, in the model of Coale and Mac-Neil, present the peculiarity of being “hidden” because of their difficulty to be delineated in data collection (Coale & McNeil, 1972; Coale, 1977). This model can be considered as a kind of Erlang process (Cox, 1962) in the sense that marriage can occur only if persons cross all of these hidden different stages before being married.

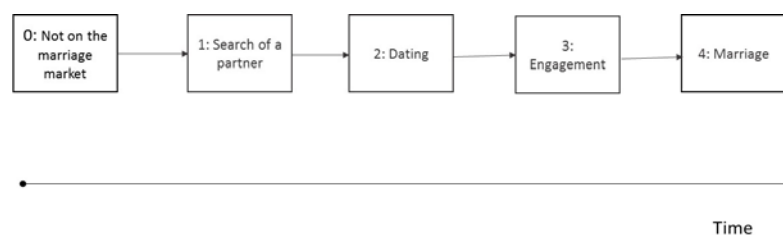


Fig. 1: The Coale and McNeil model of marriage as a compartment model

According to Coale and McNeil (1972), their model is very well suited to marriage distribution in developed countries from the Second World War to the beginning of the seventies. However, in a lot of these countries, the process of marriage and union formation transformed strongly starting in the seventies with the apparition and diffusion of extra-marital unions. Several authors proposed a sche-

ma-type of evolution of links between marriage and cohabitations in which each of these two kinds of union changed of their meaning (Villeneuve-Gokalp, 1990; Toulemon, 1997; Manting, 1996). In a first period, some precursors adopted cohabitation as an alternative to the marriage. These precursors originated from the contestation milieu in youth during the end of sixties and the beginning of seventies. They envisaged extra-marital cohabitation as an alternative to bourgeois marriage (Manting, 1996). Often, these precursors were students, and lot of them, starting from the end of seventies, became managers or exercised intellectual professions (Villeneuve-Gokalp, 1990). It is from this milieu that cohabitating unions diffused to middle and lower classes of societies. The meaning of cohabitation in the life course changed during this period of diffusion. It became a prelude to the marriage; couples experimented with living together before eventually deciding to marry. In this perspective, marriage remains an engagement to form a family. Starting from the seventies and eighties, several countries experimented with an increase in extra-marital births, which indicated that several couples no longer wished to marry when they wanted to have children. Cohabitation replaced marriage.

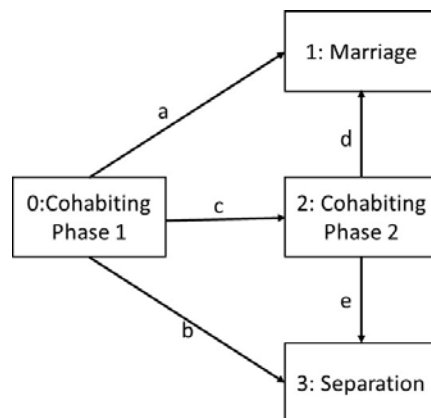
This approach, in which cohabitation is considered to have had a different succession of meanings according to its diffusion in a population, is often proposed by authors that are next to the theory of the second demographic transition (Lesthaeghe, 1995). According to this theory, demographic changes observed in developed countries since the end of the baby boom are related to the passage of an industrial to a post-industrial society. However, an alternative to this theoretical approach of cohabitation diffusing through society is present in the literature (Reiss & Lee, 1988; Perelli-Harris & Gerber, 2011). In this approach, the choice of cohabitation is constrained by economic reasons. Because marriage is expensive and because young people from lower classes often experiment with precarious jobs in the labor market, couples prefer to cohabit before marriage.

Whatever the characteristics of extra-marital unions in a country or in a social group (chosen or constrained, preceding a marriage or alternative to it), its emergence and its diffusion means a change in the process of marriage as a succession of hidden stages, as it was initially proposed in the Coale and McNeil model. In a country or in a social group in which the extra-marital union is a norm accepted by a large segment of the population, one can consider that the phase of entry on the union market is a phase that occurs before the cohabitation, as well as the phase of dating, while engagement seems to form a stage that occurs after the entry into cohabitation. This phase could have split into several phases. In this paper, we hypothesize that there are two hidden phases: a first phase corresponding to a period of trial that precedes the second phase, which can be considered as a "true" period of engagement. During this second phase of engagement, couples decide to marry, if the cohabiting union is a prelude to marriage, or remain living in cohabitation, if cohabiting union has the meaning of an alternative to the marriage. This conception of cohabiting unions into two hidden phases is the base of the model we propose to develop.

3 A compartment model for analyzing cohabitation duration

Manting (1996) and Toulemon (1997) envisaged that the hazard rate of marriage of a cohabiting couple was related to the meaning of cohabitation. In this paper, we will enlarge this point of view by: 1) taking into account that a cohabiting union can lead not only to a marriage but to a separation; and 2) supposing that the shape of the distribution of marriage and separation across the time is related to the social meaning of a cohabiting union.

We propose to develop a compartment model in which a cohabiting union is separated into two hidden phases. From each of these two phases, there is the possibility to end this cohabiting union with a marriage or a separation (figure 2). There are three possible events from the first phase of the cohabiting union: marriage, separation, and transition to the second phase of cohabiting union. From the second phase, there is the possibility to experiment a marriage.



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Fig. 2: phase-type model of cohabitation duration

The difficulty with such a model is that durations in the first phase and in the second phase are, by definition, non-measured. We do not have any information that allows us to know when a person is in the first phase or in the second phase of a cohabiting union. The only information we can generally collect in demographic surveys is the duration of each cohabiting union of respondents and how this unions ended (if it ended), with a marriage or a separation. By hypothesis, we will consider that each transition rate is time constant across the time, which means that the process is Markovian. Transition hazard rates of marriage from the first and the second phases will be denoted a and d , those of separation, b and e , while the transition rate from the first to the second phase will be denoted c (figure 2). As there is the possibility to experiment with both events from each stage of the cohabiting union, the proposed model belongs to the family of the Coxian models (Aalen, 1995).

A matrix approach allows for estimating the cumulative distribution functions (CDF) of transition to marriage and separation (Aalen, 1995; Lindqvist & Amundrustad, 1999; Lindqvist, 2013). Suppose a transition Markovian time infinitesimal matrix, in which each line i and each column j represent the different states possible. In our case, this transition matrix M can be represented by:

$$M = \begin{bmatrix} -a - b - c & c & a & b \\ 0 & -d - e & d & e \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

0 means that there is no transition possible from the state i to the state j , or on the diagonal of the matrix that any individual can leave the state i . It has been shown that the exponential of a phase-type model matrix times t gives the set of cumulative distribution function from one phase to another phase (Neuts, 1981). In the present case:

$$\exp(tM) = \begin{bmatrix} P_{00}(t) & P_{01}(t) & P_{02}(t) & P_{03}(t) \\ 0 & P_{11}(t) & P_{12}(t) & P_{13}(t) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $P_{ij}(t)$, $i \neq j$, represents the cumulative probability to have transited from phase i to phase j at time t , while $P_{ii}(t)$ represents the survival probability in phase i at time t . In the present case, the cumulative distribution functions $P_{02}(t)$ and $P_{03}(t)$ are of most interest for us, as they represent the cumulative probabilities at time t from the beginning of the union at time t_0 for, respectively, the marriage and separation. We used possibilities of symbolic computations implemented in the software Mathematica to develop formulas of each cumulative distribution function starting from the matrix M (Aalen, 1995; Wolfram Alpha, 2016). The basic hypothesis of a constant hazard rate of transition from state i to state j can be considered very simplistic, as it means that a transition is without memory of the past, at least of the duration of the first phase of the cohabiting union. Even if the starting matrix is simple, formulas of cumulative distribution of marriage $P_{02}(t)$ and separation $P_{03}(t)$ appear to be quite tedious¹:

¹ As an alternative, cumulative distributions of marriage and separation can be written with a matrix equation (Lindqvist, 2013). This alternative form presents the advantage to be flexible in a sense that a large family of phase-type models can be written by this way. However, the process of estimation of such general models in which there is the necessity to compute numerically the matrix expo-

$$\begin{aligned}
P_{02}(t) &= -\exp(-(d+e)t) * c * \frac{d}{((a+b+c-d-e) * (d+e))} + \frac{a*d+c*d+a*h}{((a+b+c) * (d+e))} \\
&\quad + \exp(-(a+b+c)t) * \left(\frac{-(a+c)*d-a*e}{((a+b+c) * (d+e))} + c * \frac{d}{((a+b+c-d-e) * (d+e))} \right) \\
P_{03}(t) &= -\exp(-(d+e)t) * c * \frac{e}{((a+b+c-d-e) * (d+e))} + \frac{b*d+c*e+b*h}{((a+b+c) * (d+e))} \\
&\quad + \exp(-(a+b+c)t) * \left(\frac{-(d+e)*b-c*e}{((a+b+c) * (d+e))} + c * \frac{e}{((a+b+c-d-e) * (d+e))} \right)
\end{aligned}$$

The sojourn function in cohabiting union $S(t)$ (in phase 1 or in phase 2) can then be computed by:

$$S(t) = 1 - P_{02}(t) - P_{03}(t)$$

Despite the complexity of formulas, such an approach with two hidden phases during the cohabitation allows envisaging different meanings of cohabiting unions, according to the level of initial hazard rates. We propose here four simulated and non-exhaustive scenarios, each of them differing by their shape of cumulative distribution of marriage and separation (table 1 and figure 3).

In the first scenario, hazard rates of marriage and separations are high compared to the transition to the second phase (hazard rate of marriage being higher in this example than the one of separation), while they become low when couples enter the second phase of cohabiting unions (figure 3, type 1). Such a set of hypotheses about the different hazard rates would correspond to a type of cohabiting union in which remaining in this union after a trial period corresponding to the first phase of cohabiting unions is scarce. In this case, the model is very similar to an exponential model with two competing events (Aalen et al., 2008). Cumulative distributions of marriage and cohabitation increase very quickly at the beginning of the process. This increase is much slower in the scenario of a higher hazard rate of hidden transition from the first phase of cohabiting union to the second one and the same level of hazard rates of marriage and separation from the first and second phases of cohabiting unions (figure 3, type 2). Such a scenario would correspond to a model of cohabiting unions as an alternative to the marriage in which a lot of couples after a trial period decide to remain cohabitant (transition to the second phase), marriages and separations becoming rare. In this case, processes of marriage and separation are slowed down by the fact that many couples enter the second phase of cohabiting unions. Processes are not terminated at the end of the period of observation—because of marriages and separations that occur when couples are in the second phase of cohabiting union—even if hazard rates are low

mental is very long at the opposite of the estimations of the model as it is expressed by this equation.

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during this second phase. A third scenario is opposed to the first one (figure 3, type 3). It corresponds to a model in which there is a trial period during the first phase of cohabiting union in which marriages as well as separations are scarce. After this trial period, there is a transition to the second phase of cohabiting union, in which hazard rates of marriage and separations become high. In this case, during the first period in which a majority of couples are yet in the first phase of cohabiting union, the cumulative distribution for each event increases slowly and then increase more quickly as soon as couples enter the second phase of cohabiting unions. The fourth type corresponds to a scenario in which the first phase of cohabiting unions correspond to a trial period, during which a lot of couples separate (figure 3, type 4). This trial period ends when couples enter the second phase of cohabiting union from which a lot of couples marry. In such a scenario, the cumulative distribution of separation is higher than the one of the marriage while a majority of couples are in the first phase of cohabitation. But when the remaining majority of cohabiting couples have transited to the second phase of the cohabiting union, the cumulated proportion of marriage can become higher than the one of separation.

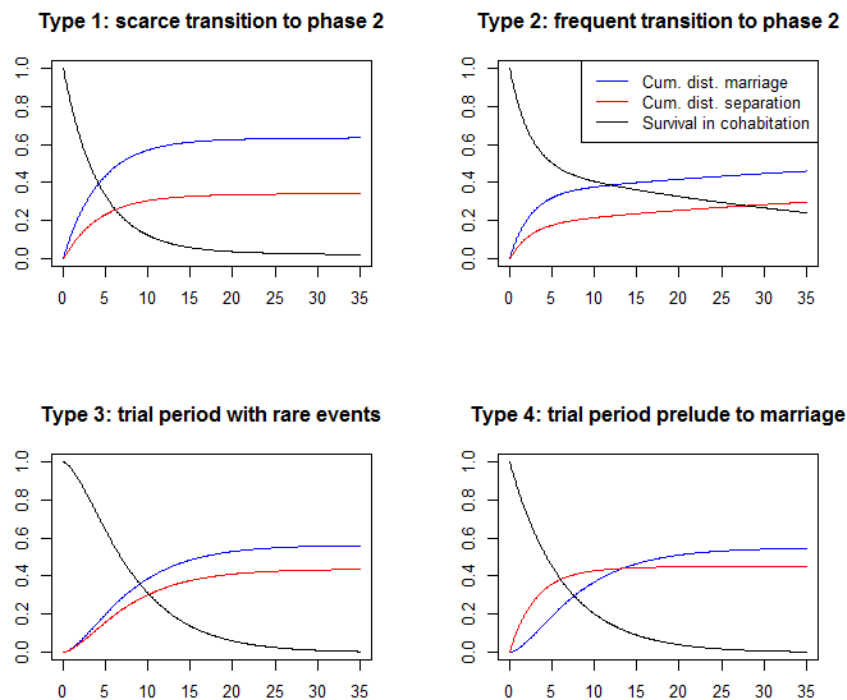


Fig. 3: Different scenarios of the role of cohabiting unions

Table 1: Different scenarios of hazard rates of the model of cohabitation

	Type 1	Type 2	Type 3	Type 4
a	0.15	0.15	0.01	0.01
b	0.08	0.08	0.01	0.15
c	0.01	0.20	0.20	0.20
d	0.01	0.01	0.20	0.15
e	0.01	0.01	0.15	0.01

Other scenarios can be envisaged, such as the opposite of the fourth type in which the hazard rates of marriage and separations are respectively high and low during the first phase of the cohabiting union, and the inverse during the second phase. However, all scenarios show that the rate of hidden transitions from the first phase to the second one plays a major role in the shape of cumulative distributions of marriage and separation.

We then draw several ideal types of cohabiting unions with two hidden phases. It becomes interesting to test the fit of the model on real data. Which scenario of cohabiting union corresponds to reality?

4 Data and model estimation

We estimated our model on first cohabiting union duration in Great Britain. Data we used come from two very similar survey databases: the National Child Development Study (NCDS) and the 1970 British Cohort Study (BCS70). The NCDS is a panel survey that interviews more or less regularly men and women ($N \approx 17\,000$) that were born on a specific week of March 1958 (CLS, 2015). Several topics have been developed in questionnaires of the different waves, often in relation to the phase of the life course of respondents: early development, childhood, health transition to adulthood, professional career, and unions and marriage. The BCS70 follows persons born during a specific week of 1970 with aims very similar to the ones of the NCDS. These two surveys are managed by the Center for Longitudinal Studies (CLS) in London. Data of these two surveys are downloadable from the website of UK data service, the data archive center in the UK (UK data service, 2015).

Among all data files available, we specifically use for each survey a file on the history of unions that was built by the CLS team (Hancock, 2011a, 2011b). In these data files, union spells of each respondent were recorded with information about the type of union at its beginning (marriage or cohabiting union) and, in the case of a cohabiting union, the outcome of this union (marriage, separation, or remaining cohabitant at the last interview). We then selected all first unions that began with a cohabiting union. We computed durations from the beginning of the union to marriage or to separation for respondents that end their union with one of these events. For those who did not experiment with any of these events during

their observation, censored duration taken into account was the time between the beginning of the union and the last interview. We also censored the rare cases of death of the partner during the cohabiting union at the moment of the death. Durations were computed originally in months, but we converted them to years. Distributions of events and censoring in each cohort for men and women are reported in table 2.

Table 2: Events and censoring in each survey (in %)

	Marriage	Separation	Censoring	N
Cohort 58 men	67.06	7.02	25.92	2878
Cohort 58 women	68.72	5.58	25.70	2436
Cohort 70 men	50.37	32.96	16.67	4648
Cohort 70 women	54.59	32.10	13.31	4825

For the sake of simplicity, we will identify these two databases as the 1958 cohort and the 1970 cohort. To be born in 1958 means that those who experiment with a cohabitation union in the seventies were akin to early adopters. Forty-five percent of first unions in this cohort were cohabiting unions at their beginning, without significant differences between men and women. Other unions were direct marriages and are not taken into account in our investigations. This proportion reaches 85% in the 1970 cohort, which means that the cohabiting union became common for people of this cohort. We can then expect that the meaning of a cohabiting union will differ between the two cohorts. In the case of cohort 58, we suppose that the cohabiting union could have signified a prelude to marriage or an alternative to the marriage. In the case of the cohort 1970, it could have signified a trial period.

For each survey, we estimated the five parameters by the likelihood maximization method. In the case of a single event, each individual contributes to the likelihood by the density $d(t)$ (which is the derivative of the cumulated distribution function of the event) if she experimented with the event and by the survival function $S(t)$ if she did not. In the present case of competing events, the likelihood equation takes into account the density of the distribution of each event $d_{02}(t)$ and $d_{03}(t)$. These two densities are the derivatives of respectively $P_{02}(t)$ and $P_{03}(t)$. Despite the complexity of formulas for cumulative distributions, their derivatives are not complicated to compute. The likelihood equation can then be written as:

$$L(a, b, c, d, e) = \prod_{i:\delta_i=2}^n d_{02}(T_i) \prod_{i:\delta_i=3}^n d_{03}(T_i) \prod_{i:\delta_i=0}^n S(T_i)$$

where n represent the total of individuals; $\delta_i = 2$ if the individual is married, $= 3$, if he is separated, and $= 0$ if the duration is truncated. T_i represents the duration associated with an individual. As usual, this is in fact the logarithm of this equation that is maximized. In order to avoid estimating negative hazard rates, we in fact will estimate logarithms of each hazard rate.

5 Results

Parameters were estimated in each cohort for men and women. We first estimated in each sub-population Aalen-Johansen estimators of the cumulative distribution of marriage and separation and survival function in cohabiting unions for men and women of each cohort (figure 3). Aalen-Johansen estimation is a non-parametric method that generalizes the Kaplan-Meier estimator in cases of competing event and multi-state models (Aalen & Johansen, 1978). It computes the cumulative distribution for each transition and the survival function in each “visible” state. In the present case, the cumulative distribution functions of marriage and separation as well as the “survival” function in a cohabiting union are estimated. The package *etm* in R was used to estimate these non-parametric estimations (Alignol et al., 2011). Results show that separations are rare in the 1958 cohort for men as well for women, while the marriage is more important for both sexes. A large part of cohabitations remain in this kind of union. In the case of the 1970 cohort, cumulative distributions of these two event were more balanced, even if marriages remained more important than separations. Remaining in cohabiting unions is less important in proportion at the end of the window of observation than in the case of the 1958 cohort.

Afterwards, we estimate our phase-type model of cohabiting unions on each sample. We used the command *mle* in the library *stats4* in R to make likelihood estimations of the five parameters of the model. Results show very good fits of the model on each sample, which gives good reasons to validate the hypothesis of a cohabiting union divided into two phases (figure 4).

A view of estimated coefficients allows for understanding processes of marriage and separations (table 3). In the case of the 1958 cohort, the hazard rate of transition from the first phase of cohabiting union to marriage and to the second phase are high, while the transition to separation is rare. When persons are in the second phase, marriages become rare while separations remain scarce. Such results correspond well to the second scenario that we proposed (figure 3 and table 2), except that during the first phase, separations are rather rare. Cohabiting unions seem to have two roles: first a prelude to the marriage and, second, an alternative to the marriage. Processes are very similar between men and women.

In the case of the 1970 cohort, transitions to marriage and even to separation are high, while the transition to the second phase became scarcer, in comparison to the 1958 cohort (table 3). When people are in this second phase, marriages and separations are rare. It corresponds more to the first ideal type of cohabiting union that we proposed (figure 3 and table 2). As in the first cohort, results show two types of cohabitations. The first one is a trial period before the marriage, in which couple can marry or break. The second cohabiting union, corresponding to the second phase, is an alternative to the marriage. As in the 1958 cohort, there are no differences in behaviors between men and women.

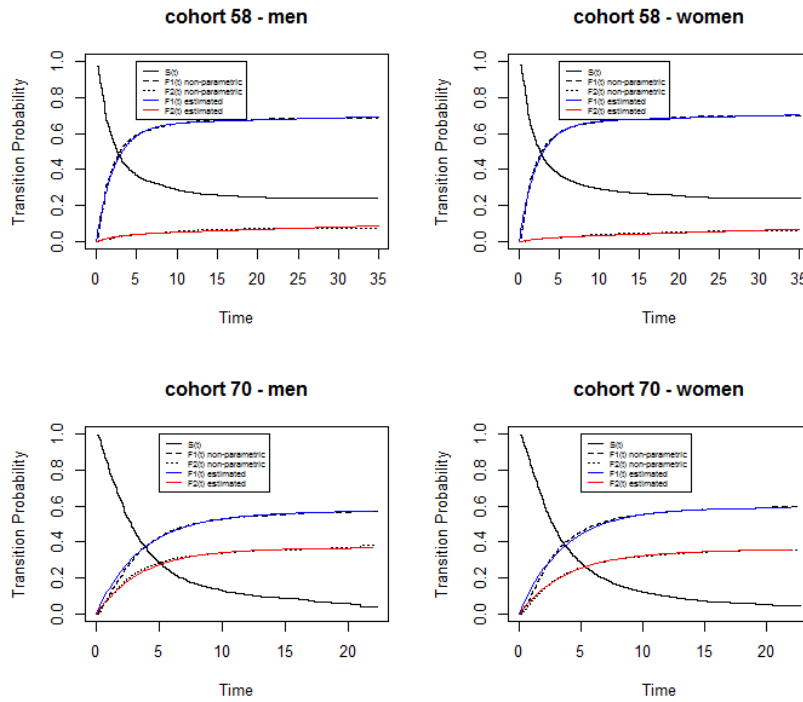


Fig. 4: Fit of estimated coefficient

Table 3: Estimated coefficients and hazard rate of each transition

		Cohort 58							
		Men			Women				
log(hazard)	Estimations	Std.error	hazard rate	log(hazard)	Estimations	Std.error	hazard		
a	-1,23	0,03	0,293	a	-1,18	0,03	0,308		
b	-4,01	0,11	0,018	b	-4,58	0,17	0,010		
c	-1,98	0,05	0,138	c	-1,92	0,05	0,146		
d	-5,35	0,20	0,005	d	-5,37	0,18	0,005		
e	-5,19	0,13	0,006	e	-5,20	0,12	0,006		
		Cohort 70							
log(hazard)	Estimations	Std.error	hazard rate	log(hazard)	Estimations	Std.error	hazard		
a	-1,85	0,02	0,157	a	-1,80	0,02	0,166		
b	-2,27	0,03	0,103	b	-2,37	0,03	0,094		
c	-3,62	0,18	0,027	c	-3,77	0,18	0,023		
d	-4,10	0,55	0,017	d	-5,27	1,26	0,005		
e	-4,86	0,65	0,008	e	-3,80	0,32	0,022		

6 Conclusion

A phase-type approach of cohabiting union duration allowed us to propose some scenarios of links between distribution of union duration and the social meaning of these unions. The estimation of the model on real data permitted to understand the evolution of this kind of union in the British context. At the opposite of a more traditional approach in event history or survival analysis in social sciences, especially in life course research, we emphasized our purpose on the shape of the hazard rate while we neglected the effect of individual characteristics on the hazard rate, as it is often proposed in the literature *via* the hypothesis of the proportionality of the hazard rate, such as in the Cox model. Expectations about effects of individual characteristics on each hazard rate, however, also can be developed in a phase-type model by hypothesizing, for example, that each hazard rate depends on these characteristics, as well as an estimating coefficient associated with each of these characteristic. The classical proportionality assumption can be called upon for estimation of the effect of characteristics on the different hazard rates. Note that, in the present case, we just further tested our model, not only in distinguishing men and women, but also by differentiating them by their age at the beginning of the union or by the social class (observed by the profession of the father or the mother of the respondent). Results obtained did not reveal significant differences between categories.

In the first section of this paper, we indicated in our review of the Coale and McNeil model that phases crossed by persons during the process leading to marriage were difficult to delineate in a quantitative survey. It is also the same in the case of the two phases we envisaged in our example of cohabiting union. The notion of hidden or unobserved phases, however, requires some verification in order to avoid mistakes. At best, the development of a phase-type model and its fit to duration quantitative data should be accompanied by other investigations, such as qualitative interviews, in order to verify the existence and the meaning of the hidden phases.

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